Model Sample Paper 2016-17 (Annual Examination)

Class – 12 (CBSE)

Subject: Mathematics

Timing: 3 Hours] Set-2 [Maximum Marks : 100

General Instructions:

(i) *All questions are compulsory.*
(ii) *The question paper consists of 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of one mark each, Section B comprises of 8 questions of two marks each, Section C comprises of 11 quest ions of 4 marks each and Section D comprises of 6 questions of 6 marks each.*
(iii) *A questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
(iv) *There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*
(v) *Use of Calculators is not permitted.*

सामान्य निर्देश –

(i) सभी प्रश्न अनिवार्य हैं |

(ii) इस प्रश्न-पत्र में 29 प्रश्न हैं जो कि चार खण्डों में विभाजित हैं – अ, ब, स तथा द | खण्ड 'अ' में 4 प्रश्न हैं जिनमें से प्रत्येक एक अंक का है | खण्ड 'ब' में 8 प्रश्न हैं जिनमें प्रत्येक दो अंक का है | खण्ड 'स' में 11 प्रश्न हैं जिनमें से प्रत्येक 4 अंक का है व खण्ड 'द' में 6 प्रश्न है जिनमें प्रत्येक 6 अंक का है |

(iii) खण्ड 'अ' में सभी प्रश्नों के उत्तर एक शब्द, एक वाक्य अथवा प्रश्न की आवश्यकता अनुसार दिए जा सकते है |

(iv) पूर्ण प्रश्न-पत्र में विकल्प नहीं है फिर भी चार प्रश्नों में तथा छः अंकों वाले दो प्रश्नों में आंतरिक विकल्प हैं | ऐसे सभी प्रश्नों में से आपको एक ही विकल्प करना है |

(iv) कैलकुलेटर के प्रयोग की अनुमति नहीं है |

SECTION-A (1 Mark)

Q1. If F: R $\rightarrow$ R defined by f (x) = $\frac{x-1}{2}$ find (fof) (x).

Q2. Find a unit vector perpendicular to both the vectors $\hat{\imath}$ - 2$\hat{\jmath}$ + 3$\hat{k}$ and $\hat{\imath}$ + 2$\hat{\jmath}$ - $\hat{k}$.

Q3. Write the value of Δ , if Δ = $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$

Q4. If $\tan^{-1} x \times \tan^{-1} y = \frac{4\pi}{5}$. find $\cot^{-1} x + \cot^{-1} y$.

SECTION-B (2 Marks)

Q5. If A = $\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$ show that A' A^{-1} = $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$.

Q6. Evaluate $\int x\,(\log x)^2\,dx$.

Q7. If y = $\sqrt{\frac{1+e^x}{1-e^x}}$ show that $\frac{dy}{dx} = \frac{e^x}{(1-e^x)\sqrt{1-e^{2x}}}$.

Q8. If A and B are two events such that that p(A) = $\frac{1}{4}$, P (B) = $\frac{1}{2}$ and

P (A $\cap$ B) = $\frac{1}{8}$ find P(not A and not B)

Q9. Simplify: [sin $2\,tan^{-1}\sqrt{\frac{1-x}{1+x}}$].

Q10. Verify LaGrange's mean value theorem for function f(x) = (x - 3) (x - 6) (x - 9) on the interval [3, 5]

Q11. Find the vector equation of the line passing through the point (2, 3, 2) and parallel to the line $\vec{r}$ = (-2$\hat{\imath}$ + 3$\hat{\jmath}$) + λ(2$\hat{\imath}$ - 3$\hat{\jmath}$+ 6$\hat{k}$). also find the distance between the lines.

Q12. Find the approximate value of f(2.01) where f(x) = $4x^2$+5 x +2

SECTION-C (4 Marks)

Q13. Using elementary transformation, find the inverse of the matrix A = $\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

Q14. If y = x^x prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x}$ = 0.

Or

The volume of a cube is increasing at a constant rate. Prove that the increase in its surface area varies inversely as the length of an edge of the cube.

Q15. Evaluate: $\frac{dx}{\sin x - sin2x}$.

Q16. Find all point of discontinuity of f where f(x) = $\begin{bmatrix} \frac{sinx,}{x} & if\ x < o \\ x + 1, & if\ x \geq o \end{bmatrix}$

Q17. Find the difference between the greatest and least value of the function f(x) = sin $2x$ - x on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Q18. Find the equations of the tangent and normal to the curve x = a $\sin^3\theta$ and y = a $\cos^3\theta$ at θ = $\frac{\pi}{4}$.

Q19. Find the shortest distance between the lines whose vector equations are

$\vec{r}$ = (1 – λ) $\hat{\imath}$ + (λ – 2) $\hat{\jmath}$ + (3 – 2λ) $\hat{k}$

$\vec{r}$ = (μ – 1) $\hat{\imath}$ + (2μ – 1) $\hat{\jmath}$ + (2μ + 1) $\hat{k}$

Q20. Show that the vector's $\vec{a}$ = 3$\hat{\imath}$ -2$\hat{\jmath}$+ k, $\vec{b}$ = $\check{\imath}$-3$\check{\jmath}$+$\hat{k}$, $\vec{c}$ = 2$\hat{\imath}$ + $\hat{\jmath}$-4$\hat{k}$ from a right angled triangle.

Q21. A committee of 4 students is selected at random from a growth consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee given that at least one girl must be there in the committee.

Q22. Two schools A and B decided to award prize to their students for three values honesty (x), punctuality (y) and obedience (z). School A decided to award a total of ₹ 11000 for the values to 5, 4, and 3 students respectively while school B decided to award ₹ 10700 for the three values to 4, 3 and 5 students respectively. If all the three prizes together amount to ₹ 2700 then -

(i) Represent the above situation by a matrix equation and form linear equation using matrix multiplication.

(ii) Is it possible to solve the system of equation so obtain using matrices ?

(iii) Which value you prefer to be rewarded most and why?

Q23. A random variable x has the following probability distribution:

x	0	1	2	3	4	5	6
P(x)	C	2C	2C	3C	C^2	$2C^2$	$7C^2$+C

Find the value of C and also calculate mean of the distribution.

SECTION - D (6 Marks)

Question numbers 24 to 29 carry 6 marks each.

Q24. Draw rough sketch of the region and find it's area using integration.

(x, y) : $y^2 \le$ bax and $x^2 + y^2 \le 1ba^2$.

Q25. Evaluate:

$$\int_o^{\pi} \frac{xdx}{(a^2\ cos^2\ x + b^2 \sin^2 x)^2}$$

Or

Evaluate the integral as a limit of sum:

$\int_0^2 e^{3-2x}$ dx

Q26. Solve for x $tan^{-1}\left(\frac{x-2}{x-1}\right) + tan^{-1}\left(\frac{x+2}{x+1}\right) = \frac{\pi}{4}$

Or

A binary operation $*$ is defined on the set x = R – $\{-1\}$ by x $*$ y = x + y + xy, ∀x, y ∈ x

Check whether $*$ is commutative and associative. Find its identity element and also find the inverse of each element of x.

Q27. Using properties of determinate prove the following.

$$\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

Q28. A company manufactures there finds of calculators: A, B and C in its two factories I and II. The company has got an order for manufacturing at least 6400 calculators of kind A, ₹ 4000 of kind B and 30 of kind C. The daily output of factory I is of 50 calculators of kinds A, 50 calculators of kind B and 30 calculators of kind C. The daily output of factory II is of 40 calculators of kind A, 20 to kind B and 40 of kind C. The cost per day to run factory I is ₹12000 and of factory II is ₹15000. How many days do the two factories have to be in operation to produce the order with the minimum cost? Formulate this problem as an LPP and solve it graphically.

Q29. Find the distance of the point (–1, –5, –10) from the point of interaction of the line $\vec{r} = \left(2\hat{\imath} - \hat{\jmath} + 2\hat{k}\right) + \lambda\left(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{\imath} - \hat{\jmath} + \hat{k}\right) = 5$.

*****The Best Of Luck ******